Boundary-imposed spiral drift

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(Received 21 November 1995)

The spiral wave behavior was examined in a bounded medium within the framework of the Belousov-Zhabotinsky reaction. When the spiral tip was located far from the center of a circular medium, the spiral wave was observed both to drift along the boundary and to approach it. [S1063-651X(96)06305-2]

PACS number(s): 05.70.Ln, 82.20.Mj

I. INTRODUCTION

Spiral waves constitute an example of self-sustained activity that has been observed in various excitable media such as cardiac muscle [1], cultures of the slime mold *Dictiostelium-Discoideum* [2], or oscillating chemical reactions, such as the Belousov-Zhabotinsky (BZ) reaction [3]. The study of spiral properties, which rarely depend on the medium that sustains them, has constituted an important challenge to the scientists for decades, so considerable effort has been devoted experimentally [4,5], numerically [6,7], and theoretically [8–10] to the understanding of such structures.

Despite that these previous studies provide an important background on spiral properties, little is known about the interaction between the waves and the medium where they spread. Only recently, some authors have turned their attention to this phenomenon, although from different points of view. Davidenko *et al.* [11] observed drifting spiral waves in isolated cardiac tissue and Biktashev and Holden [12] described a defibrillation method in which they considered the boundary effects, in addition to an external forcing. On the other hand, Davydov and Zykov [13] considered this effect in small media where the length of the front was comparable to the extent of the medium. They concluded that a rigid rotation can become unstable when the spiral tip is initially displaced from the center of the medium. This prediction was corroborated by the experiments carried out by Müller and Zykov [14] in a small Petri dish using the Belousov-Zhabotinsky reaction [3]. Besides, Sepulchre and Babloyantz [15,16] studied spiral motion in a small medium—a few wavelengths—for a system near the Hopf bifurcation and with relaxation oscillation in square and circular geometries. Finally, other authors proved theoretically [17,18] that rigid rotation is not generic. They showed that the interaction with a boundary (qualitatively similar to the interaction with other spirals [19] or with defects [20]) gives rise to localized deformations, small in comparison with the spiral wavelength and, consequently, to the drifting of a spiral as a whole.

The purpose of this paper is to investigate experimentally how the boundaries induce the drift of a rotating vortex in a round medium when a vortex is initially displaced from the geometrical center of the medium.

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II. MATERIALS AND METHODS

The experiments were performed in a BZ reaction, where the catalyst (ferroin) was immobilized in a *silica gel* [21] at room temperature (25.0±1.0 °C). A 1 mm thick gel was initially prepared in a Petri dish 88 mm in diameter and cut in a round shape (2.4 mm in diameter) to avoid capillarity effects close to the Petri dish walls. During the experiments, the gel layer was covered with a thick liquid layer (1 cm) of the other BZ reagents [$NaBrO_3$, 0.17*M*; H_2SO_4, 0.17*M*, and $CH_2(COOH)_2$, 0.17*M*] to prevent any interaction between the reaction and the oxygen in the air. In order to observe any appreciable displacement in the tip position, it was necessary to perform experiments lasting several hours and, as the reagents properties vary in time, their concentrations were maintained constant along the experiments by imposing a flow of reagents (100 cm^3/h) into the Petri dish. We have used three tanks with the three liquid reagents of the Belousov-Zhabotinsky reaction to avoid chemical reaction before reaching a mixing chamber. After surpassing that chamber, the reagents flowed into the Petri dish near its walls to prevent the appearance of chemical gradients close to the silica gel. On the other hand, the flow was uniformly distributed along the Petri dish wall to avoid directional changes in chemical concentration.

The experiments were started 15 minutes after pouring the liquid over the gel layer in order to obtain a homogeneous concentration into the bulk. A spiral wave was generated as follows: the medium was stimulated at a certain point by touching the gel with a silver wire [22] which gave rise to the spreading of a circular wave from that point. Two discontinuous wave fronts were generated either by inhibiting a part of the front with a piece of iron or by vulnerability [23].

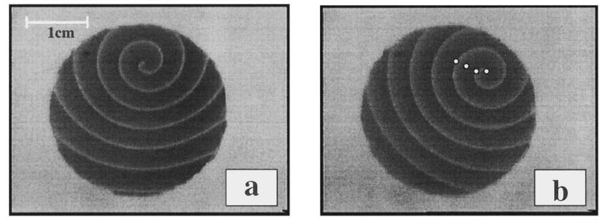

FIG. 1. Drift of a spiral wave toward the boundaries. A spiral wave is plotted at different times during the experiment; (a) 1 h, (b) 5.5 h. It can be observed that the spiral has drifted toward the closest boundary. The lighter spots in image (b) represent the tip position at different times (1.0, 2.5, 4.0, and 5.5 h after the beginning of the experiment, respectively).

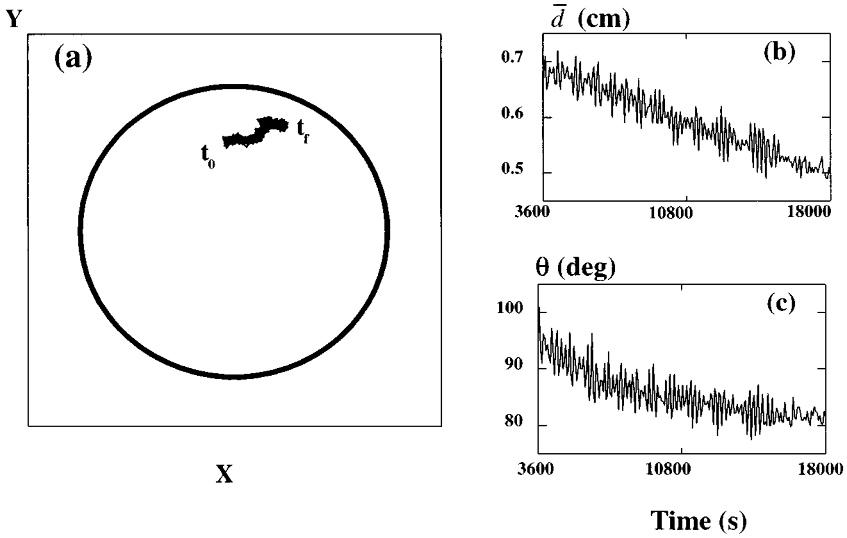

FIG. 2. Trajectory followed by the spiral tip. The tip movement corresponding to the images in Fig. 1 are plotted in (x,y) coordinates in (a), where t_0 corresponds to the initial position of the tip and t_f to the final one. In polar coordinates, in (b) the radial distance d to the closest boundary decreases with time and the phase (c) decreases due to the clockwise rotation of the spiral tip around its core. The rotation direction of the drift coincides with the tip rotation.

Once both spirals were generated, one of them was removed from the medium and the other one was initially placed at a given position by means of a parallel electric field [24,25].

Throughout the experiments, the spiral tip was followed by a charge coupled device (CCD) camera and recorded on a video tape for further analysis of its movement. The tip position was automatically measured every five seconds.

III. RESULTS

When a spiral was created at a certain distance from the center of the Petri dish, the existence of a composite movement was observed. On the one hand, the spiral tip described a circular movement, similar to that one described by a spiral in an unbounded medium and, on the other hand, the spiral tip was observed both to drift along the boundary and to approach it. This drift can be observed in Figs. 1(a) and 1(b), which correspond to 1 and 5.5 h after the beginning of the experiment, respectively. Note that, as was previously mentioned, the medium properties were controlled in such a way that both the spiral period and its wavelength remained constant ($T=110\pm5$ s, $\lambda=2.75\pm0.25$ mm) during the experiments—lasting six hours.

In Fig. 2(a), one can observe the spiral tip movement in (x,y) coordinates. In Figs. 2(b) and 2(c) the same movement is plotted in polar coordinates as a function of time, where the distance from the spiral tip to the center of the circular medium is shown to decrease with time, i.e., the spiral approaches the boundary. On the other hand, the phase is observed to decrease during the experiment due to the clockwise rotation of the spiral. In every experiment, the phase displacement was observed to depend on the spiral chirality, in such a way that clockwise (counterclockwise) spirals drift in the clockwise (counterclockwise) direction. We have observed that the spiral chirality only changes the sign of the rotation velocity, but not its module, which remains unchanged for a constant distance between the spiral tip and the center of the medium.

When different initial radii were considered, it was possible to obtain different radial and angular velocities. In Figs. 3(a) and 3(b) both velocities are plotted as a function of the distance to the nearest boundary along the radial direction. In both figures, the velocity is observed to depend on d (mean distance to the nearest boundary calculated every hour) although in a different way. In Fig. 3(a), two regions can be observed, the first one corresponding to distances (d) longer than one wavelength. In this region, the radial velocity $(V_r=-dd/dt)$ decreases when d increases. The dashed line represents the fitting of the experimental data to an exponential function. On the contrary, there is another region, for distances smaller than a single wavelength, where the radial velocity becomes irregular. In Fig. 3(b), the phase velocity $(V_\theta=-d\theta/dt)$ is shown to increase with $\bar{d}$. Note that we have calculated a mean velocity—corresponding to a mean position—because it is not possible to calculate the drift ve-

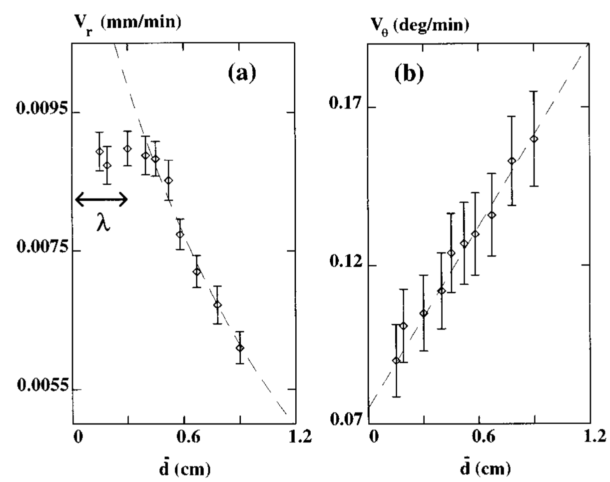

FIG. 3. Drift velocities corresponding to different distances to the boundaries. The radial (a) and angular velocities (its module) (b) are plotted as a function of the average distance to the boundary $\bar{d}$ (the radius of the medium is 1.20 cm). In (a), for $\bar{d}<\lambda$, the radial velocity behaves in an irregular way due to the proximity to an irregular edge. On the contrary, for $d>\lambda$, the radial velocity increases exponentially when d decreases—it fits to $V_r(d)=1.59\times10^{-3}+1.12\times10^{-2}\exp(-d)$ with an accuracy better than 98%. In (b) the module of the angular velocity increases with $\bar{d}$.

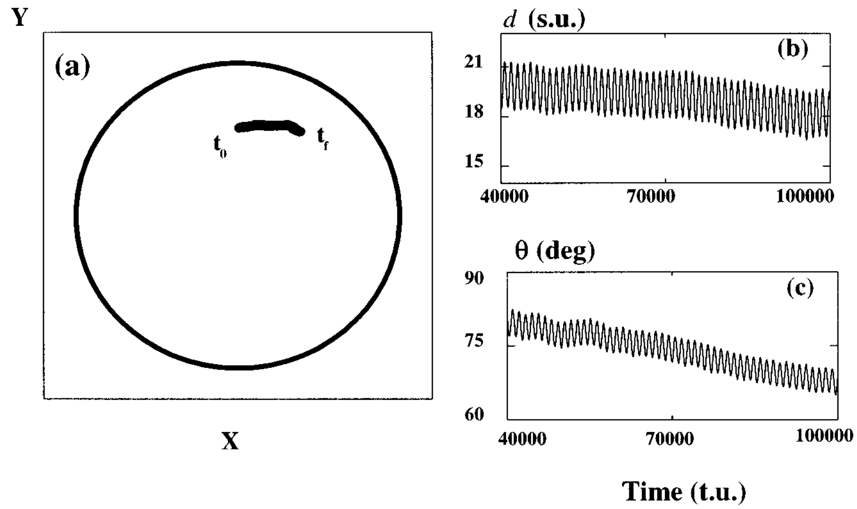

FIG. 4. Geometrical-kinematical simulation of spiral drift. By means of the geometrical-kinematical method explained into the test, it is possible to observe the spiral drift toward the closest boundary. This can be observed in Cartesian (a) and in polar coordinates [(b) radial coordinate and (c) angular coordinate]. Note that the behavior is qualitatively similar to the one shown in Fig. 2. [Parameters $V^{\infty}(0)=1$, $G^{\infty}=0.5$, $\alpha=0.5$, $\beta=0.1$, $\omega=1.0$, and $R_{\max}=50$ spatial units). t.u. denotes time units.

locity at every instant, since the order of magnitude of the drift is not much larger than the spiral tip movement due to the tip rotation around the spiral core and, thus, that rotation cannot be neglected.

IV. DISCUSSION

In the preceding section, we have observed the different behavior of V_r depending on the distance to the nearest boundary. Next to the boundary ($d<\lambda$) the tip movement became irregular since, in spite of the gel was carefully cut by means of a sharp-edged cylinder, the border of the gel was not completely regular. Thus for different experiments, the spirals were observed to drift along the boundary at a constant distance from it or to collide with the borders of the gel where they were annihilated.

As for the region far from the boundaries ($d>\lambda$), the dependence of the radial velocity on d has shown to be regular (V_r decreases exponentially when d increases as predicted by other authors [12]). Nevertheless, other functions like d^{-x} with $x\in\mathbb{R}$ (in particular $x\approx 0.5$) showed to fit correctly the experimental data.

Note that our experimental results cannot be compared with those obtained theoretically by Davydov and Zykov and Mikhailov [13,26], who assumed a linear approximation only valid for a size of the medium comparable to the length of the front. On the other hand, our data cannot be easily compared with those obtained numerically by Sepulchre and Babloyantz [15] either—the disappearance of spirals when initially placed near the boundary was the only phenomenon observed in both approaches. They considered a medium whose size did not exceed a few wavelengths and found different trajectories described by the spiral tip depending on the initial position of the vortex, the geometrical shape of the system, and the relaxational character of the oscillations. The ratio R/λ (R: radius of the disk; λ: spiral wavelength) is bigger in our experiments than in their simulations and they showed that for different radii, different trajectories can be obtained. Nevertheless, they observed the spiral approach to the boundary before reaching a stationary trajectory—circular or with regular loops. Our experimental results show the same transient to those trajectories, but our experiments lasting six hours did not allow us to elucidate whether the final orbit was circular or with regular loops—to observe their asymptotical trajectories in our medium it would be necessary to have evolution times much bigger than allowed by our reactor [27,28].

In order to explain our results we have developed a simple phenomenological model that accounts for the interaction between boundaries and rotating spirals. Thus we have assumed the existence of zero flux boundary conditions in such a way that when a front arrives next to a boundary its shape is suddenly modified—it tends to become perpendicular to the border. Thus the boundary speeds up the front next to it and, consequently, the following fronts suffer this effect with a certain delay—it is well known [8,9,26] that both the normal velocity of a front and the normal and tangential velocity of a tip at a certain point depend on the time elapsed since the last excitation of that point. Thus the normal velocity of any part of a front and the tangential velocity of the tip can be formulated as follows:

$$V(\lambda)=V-f(\lambda,T_r),$$
$$G(\lambda)=G-g(\lambda,T_r), \tag{1}$$

where T_r is the characteristic recovery time of the medium, λ the distance between two consecutive fronts, V and G represent the normal and tangential velocities when the period between successive excitations T, (the distance λ) tends to infinite ($T_r/T\rightarrow 0$). Note that for a constant T_r the functions f and g increase when λ decreases, but they always remain smaller than V and G, respectively.

Keeping in mind this relation between λ and the normal velocity of the front, the sudden acceleration of a front close to a boundary increases the distance to the next one, allowing the second front to propagate faster, and so successively. If the initial position of the tip is displaced from the geometrical center of the Petri dish, the effect of the boundaries on the tip will depend on the direction, and, consequently, the tip will move faster toward the closest boundary than toward the farthest one. Note that our phenomenological model, which is similar in spirit to the kinematical-geometrical models shown in [7, 29], is only valid when the distance from the tip to the boundaries is larger than one wavelength. In this case, the front can be considered circular in a first approach when arriving at the walls and, consequently, the drift only depends on the distance to the boundaries. So, to describe the tip motion, we have considered [8,26]

$$\dot{X}_0 = -V(0)\sin(\alpha_0) - G\ \cos(\alpha_0),$$

$$\dot{Y}_0 = V(0)\cos(\alpha_0) - G\ \sin(\alpha_0), \qquad (2)$$

$$\dot{\alpha}_0 = \omega,$$

where $V(0) = V^{\infty}(0) - (\alpha/d)$, $G = G^{\infty} - (\beta/d)$, and $V^{\infty}(0)$, G^{∞} are the normal and tangential velocities in an unbounded medium and **d** the distance between the spiral tip and the closest boundary in the direction perpendicular to the tip movement. Although we have considered an interaction proportional to $(1/d)$, similar to the one given in [13], other authors have considered the existence of a much weaker interaction with the boundaries. Thus in [12] an exponential interaction was assumed and in [17] a superexponential interaction was supposed, because the previous ones were considered to exaggerate the boundaries influence. On the other hand, the screening effects from the emitted waves were neglected.

By means of this extremely simplified model, it is possible to observe [Fig. 4(a)] the existence of a drift toward the boundaries. In Figs. 4(b) and 4(c), it can be observed a movement similar to that experimentally observed [see Figs. 2(a) and 2(b)]; namely, the radial distance d to the nearest boundary decreases and there is a clockwise rotation, which corresponds to the clockwise tip movement we have considered in our calculations.

ACKNOWLEDGMENTS

This work was partially supported by Xunta de Galicia, under project XUGA-10611B93. We also want to acknowledge Professor V. A. Davydov for helpful discussions and to A. V. Holden for a critical reading of the preliminary manuscript.

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